

5.6 Fluctuations and noise

Thermodynamic fluctuations^a

Fluctuation probability	$\text{pr}(x) \propto \exp[S(x)/k]$	(5.130)	pr x S A	probability density unconstrained variable entropy availability
	$\propto \exp\left[\frac{-A(x)}{kT}\right]$	(5.131)		
General variance	$\text{var}[x] = kT \left[\frac{\partial^2 A(x)}{\partial x^2} \right]^{-1}$	(5.132)	var[.] k T	mean square deviation Boltzmann constant temperature
Temperature fluctuations	$\text{var}[T] = kT \left. \frac{\partial T}{\partial S} \right _V = \frac{kT^2}{C_V}$	(5.133)	V C _V	volume heat capacity, V constant
Volume fluctuations	$\text{var}[V] = -kT \left. \frac{\partial V}{\partial p} \right _T = \kappa_T V kT$	(5.134)	p κ _T	pressure isothermal compressibility
Entropy fluctuations	$\text{var}[S] = kT \left. \frac{\partial S}{\partial T} \right _p = kC_p$	(5.135)	C _p	heat capacity, p constant
Pressure fluctuations	$\text{var}[p] = -kT \left. \frac{\partial p}{\partial V} \right _S = \frac{K_S kT}{V}$	(5.136)	K _S	adiabatic bulk modulus
Density fluctuations	$\text{var}[n] = \frac{n^2}{V^2} \text{var}[V] = \frac{n^2}{V} \kappa_T kT$	(5.137)	n	number density

^aIn part of a large system, whose mean temperature is fixed. Quantum effects are assumed negligible.

Noise

Nyquist's noise theorem	$dw = kT \cdot \beta \epsilon (e^{\beta \epsilon} - 1)^{-1} dv$	(5.138)	w k T	exchangeable noise power Boltzmann constant temperature	
	$= kT_N dv$	(5.139)	T _N		noise temperature
	$\simeq kT dv \quad (h\nu \ll kT)$	(5.140)	βϵ v h		= hν/(kT) frequency Planck constant
Johnson (thermal) noise voltage ^a	$v_{\text{rms}} = (4kT_N R \Delta\nu)^{1/2}$	(5.141)	v _{rms} R Δν	rms noise voltage resistance bandwidth	
Shot noise (electrical)	$I_{\text{rms}} = (2eI_0 \Delta\nu)^{1/2}$	(5.142)	I _{rms} -e I ₀	rms noise current electronic charge mean current	
Noise figure ^b	$f_{\text{dB}} = 10 \log_{10} \left(1 + \frac{T_N}{T_0} \right)$	(5.143)	f _{dB} T ₀	noise figure (decibels) ambient temperature (usually taken as 290 K)	
Relative power	$G = 10 \log_{10} \left(\frac{P_2}{P_1} \right)$	(5.144)	G P ₁ , P ₂	decibel gain of P ₂ over P ₁ power levels	

^aThermal voltage over an open-circuit resistance.

^bNoise figure can also be defined as $f = 1 + T_N/T_0$, when it is also called "noise factor."

